Type-2 Fuzzy Relations: An Approach towards Representing Uncertainty in Associative Medical Relationships

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Abstract The acquisition of precise values such as symptoms, signs, laboratory test results, and diseases/diagnoses for expressing meaningful associative relationships between medical entities has always been regarded as a critical part of developing medical knowledge-based systems. After the introduction of fuzzy sets, researchers became aware of the fact that a central problem in the use of fuzzy sets is constructing the membership function values. The complication arises from the uncertainty associated with assigning an exact membership grade for each element within the considered fuzzy set. Type-2 fuzzy set handles this problem by allocating a different fuzzy set to each element. This paper addresses the subject of medical knowledge acquisition and representation by proposing consistent interval type-2 fuzzy relations in the context of fuzzy inclusion as a measure of representing the degrees of association between medical entities. The concept of interval type-2 fuzzy relation will be introduced to represent the uncertainty and vagueness between medical entities.

1 Introduction and Motivation

Associative relationships between *medical entities* such as symptoms, signs, laboratory test results, and diseases/diagnoses can be established in different ways. Medical knowledge has been formally represented by several *symbolic and/or numerical,* or *data- and knowledge-driven* methods, all of which have been used successfully to a certain extent (Fig. 4). An associative relationship between a symptom s and a disease d might be expressed in *two types* of measures: the *necessity of occurrence* of

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s with d, and the *sufficiency of occurrence* of s for d. In our context, the necessity of occurrence[1] may be interpreted as *backward implication* from d to s, i.e., $(s \leftarrow d)$, while the sufficiency of occurrence[2] may be viewed as *forward implication* from s to d, i.e., $(s \rightarrow d)$. Similarly, negative associative relationships may also be specified as backward implication or forward implication, $(s \leftarrow \neg d)$, and $(s \rightarrow \neg d)$.

The above relationships may be extended by considering multi-valued implications $\in [0, 1]$. For example,[3] the *necessity of occurrence* may be represented as multi-valued backward implication $(s \overset{\mu}{\leftarrow} d),\ with\ \mu \in [0, 1]$, and the *sufficiency of occurrence* as forward implication $(s \underset{\mu}{\rightarrow} d),\ with\ \mu \in [0, 1]$.

Human *expert knowledge* can be used to obtain values expressing *possible degrees* of uncertainty, while *statistical data* may add to the respective body of knowledge. These aspects have been successfully employed in representing medical relationships between symptoms, signs, laboratory test results and diseases in the differential diagnosis support systems CADIAG-I [1–4] and CADIAG-II [5–11]. In CADIAG-II, fuzzy set theory and fuzzy logic were used to represent the inherent unsharpness of linguistic medical terms by fuzzy sets, and to represent partial truths of medical relationships between these terms. Here, the *frequency of occurrence* and the *strength of confirmation* correspond to the *necessity of occurrence* and the *sufficiency of occurrence*. In addition, *negative associative implications* were considered to specify a *strength of exclusion*. *Semi-automatic statistical analyses* were thus able to support the knowledge acquisition process [12, 13].

However, creating a *solid medical knowledge base* is not always a straightforward process. It may be fraught with various problems, such as:

- Uncertainty: Exact values for expressing meaningful associative relationships are not easily obtained. Human expert knowledge and statistical data analyses might support this process. However, even quantitative medical information is never 100% accurate. Fuzzy systems have, in fact, superseded conventional methods in a variety of scientific applications. However, type-1 fuzzy systems, whose membership functions are type-1 fuzzy sets, are able to cope with uncertainties. Type-2

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[1] Necessity: The occurrence of symptom s is said to be necessary for a disease d, then the occurrence of d guarantees the occurrence of s; e.g., s: *"increased serum glucose level"* is obligatory for d: *"diabetes"*:

$$(s \leftarrow d)$$

[2] Sufficiency: The occurrence of symptom s is said to be sufficient for the disease d, then the existence of s guarantees the occurrence of d; e.g., s: "*the detection of intracellular urate crystals (tophi)*" confirms d: *"gout by definition"*:

$$(s \rightarrow d)$$

[3] For example: The occurrence of s: *"increased serum glucose level"* is necessary for d: *"diabetes"*, however s confirms *"diabetes"* only with 0.65:

$$(s \underset{1}{\longleftarrow} d, s \underset{0.65}{\longrightarrow} d)$$

fuzzy systems relying on type-2 fuzzy sets try to handle these uncertainties further by assigning fuzzy sets or any interval $\subset [0, 1]$, defining possibilities for the primary membership. Furthermore, representing human expert knowledge and interpreting statistical data about ignorance (what is unknown?) should also be considered. *Experts' agreements* or *disagreements and conflicts among the various sets of alternatives* concerning the relevance of data [14] play a certain role in increasing the degree of uncertainty.

- Inconsistency: Medical knowledge bases containing a large quantity of relationships among medical entities might be affected by inconsistencies and incompleteness. The quality of knowledge must be ensured by appropriate checking.

The aim of the present paper is to present the basic principles of dealing with the above-mentioned aspects of uncertainties. We focus on the following aspects:

- Employing interval type-2 fuzzy relations to handle vagueness and uncertainty.
- Employing an interval-type-2-fuzzy-relation-based inclusion measure to represent binary associative relationships between medical entities. This inclusion measure corresponds to uncertain and imprecise implication relations; i.e. binary fuzzy rules. The direction of inclusion measure corresponds to the *necessity* and *sufficiency of occurrence*, which can be represented as uncertain backward and forward implication relationships. The intervals express the *possible degrees* of inclusion of one fuzzy set in another related fuzzy set, e.g., $(s \xrightarrow[I(s,d)]{} d)$, and $(s \xleftarrow[I(d,s)]{} d)$, where $I(s, d)$, and $I(d, s) \subseteq [0, 1]$. $I(s, d)$ and $I(d, s)$ represent the *uncertainty* about *a fuzzy rule*. Here, the concept of interval type-2 fuzzy relation was adopted to reduce computational complexity.
- The knowledge base of such rules should be consistent. In other words, only relationships with consistent uncertainties expressed in the form of consistent intervals are considered.

1.1 Related Work

Interval-valued techniques have been suggested by many researchers for representing uncertainty and incompleteness. Zadeh [15] proposed type-2 fuzzy sets, whose membership functions themselves are specified by fuzzy sets. This step was necessary to consider the possible uncertainty of fuzzy set functions themselves [16–19]. Baldwin [20] proposed the assignment of necessity and possibility support boundaries to *logic programs* in order to consider *uncertainty*. Turksen [21] employed compositional operations in connection with conjunctive and disjunctive normal forms to handle approximate reasoning.

The concept of *fuzzy inclusion* has been addressed by some researchers [22–24]. It has been employed in some areas of computing, such as *image processing* and *natural language processing* [25]. Helgason and Jobe [26] focused on perception-based reasoning, utilizing medical quantities such as *necessary causal ground* and *sufficient causal ground* extracted from fuzzy cardinality. In addition, the different enhancements of the CADIAG-II medical fuzzy decision support system [6, 10, 13] are closely connected to the proposed model in the sense of considering type-1 fuzzy relations in representing the *frequency of occurrence* and the *strength of confirmation*. Another proposal is the use of *bidirectional compound* binary fuzzy rules to represent medical knowledge without applying it to type-2 fuzzy set theoretical aspects and notations [27].

Regarding the use of conditional probabilities as multi-valued implications [28], the afore-mentioned study is similar to our model in terms of considering *conditional probabilities* as a type of inclusion relationship.

In the following, theoretical definitions for type-2 fuzzy set, type-2 fuzzy relation, and interval type-2 relations will be introduced on the basis of previous reports [16, 18, 27, 29–31], as preliminaries to the concept of interval type-2 fuzzy relation describing a fuzzy inclusion.

1.2 Preliminaries: Fuzzy Sets and Relations

Definition 1 (*Type-1 fuzzy set,* $\tilde{A}$)[4] A type-1 fuzzy set, denoted $\tilde{A}$ on the referential set $\mathcal{X} = \{x_1, x_2, ..., x_n\}$ is defined as a function $\mu_{\tilde{A}} : X \to [0, 1]$, i.e., as the set of pairs:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in \mathcal{X}\}. \tag{1}$$

This function is called a membership function. $\mu_{\tilde{A}}(x)$ is the degree of membership of the element x $\in \mathcal{X}$ in the set $\mu_{\tilde{A}}(x)$. Each membership degree $\mu_{\tilde{A}}(x)$ is *fully certain*, which means that in a type-1 fuzzy set, for each x value, there is no uncertainty associated with the primary membership value.

Definition 2 (*Type-2 fuzzy set,* $\tilde{\tilde{A}}$) Based on [29, 31], a type-2 fuzzy set denoted $\tilde{\tilde{A}}$, is defined as a function $\mu_{\tilde{\tilde{A}}} : X \times [0, 1] \to [0, 1]$, i.e., as the set of triples:

$$\tilde{\tilde{A}} = \{((x, u), \mu_{\tilde{\tilde{A}}}(x, u)) | \forall x \in X, \forall u \in J_x \subseteq [0, 1]\}$$

here,

$$0 \leqslant \mu_{\tilde{\tilde{A}}}(x, u) \leqslant 1$$

For any given value of x, the $\mu_{\tilde{\tilde{A}}}(x, u), \forall u \in J_x$, is a type-1 membership function. J_x is used to reference the set of u values associated with each point in the X-axis.

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[4] Based on [29, 31].

Based on Definition 2, a membership function of a type-2 fuzzy set can be represented by its graph in 2-D space[5]:

- The primary variable or the X-axis,
- the secondary variable or the Y-axis denoted by u, and
- the Z-axis, the membership function value (secondary grade); i.e. $\mu_{\tilde{\tilde{A}}}(x, u) \in [0, 1]$.

When uncertainties are removed, a type-2 membership function reduces to a type-1 membership function; i.e. the third dimension disappears.

To reference and describe the uncertainty in the primary memberships of a type-2 set, the concept of footprint of uncertainty or FOU is defined as:

Definition 3 (*Footprint of uncertainty, FOU*) Let $\tilde{\tilde{A}}$ be a type-2 fuzzy set, then:

$$FOU = \{\mu_{\tilde{\tilde{A}}}(x, u) | (x, u) \in X \times [0, 1]\}. \tag{2}$$

We can represent FOU as the union of all primary memberships:

$$FOU(\tilde{\tilde{A}}) = \bigcup_{x \in X} J_x$$

In Figs. 2 and 6, FOU is represented as shaded regions. Here, FOU is useful to access the minimal values and maximal values of uncertainties.

In our approach to simplify the complexity involved in a type-2 fuzzy set, the concepts of interval fuzzy set and interval type-2 fuzzy relation have been adopted to represent the uncertainty.

Definition 4 (*Interval type-2 fuzzy set*) A type-2 fuzzy set is an interval type-2 fuzzy set, if for every x there exists an interval $[\underline{u}, \overline{u}]$ such that $\mu(x, u) = 1$ for all u from this interval and $\mu(x, u) = 0$ for all other u.

Definition 5 (*Type-1 fuzzy relation, $\tilde{R}$*) Let $X = \{x_1, x_2, x_3, ..., x_n\}$ and $Y = \{y_1, y_2, y_3, ..., y_m\}$ be referential sets. A type-1 fuzzy relation, denoted $\tilde{R}$ on $X \times Y$, is defined as:

$$\tilde{R} : X \times Y \to [0, 1],$$

$$\tilde{R} = \{(x_i, y_j), \mu_{\tilde{R}}(x_i, y_j)\},$$

[5] For illustration see Fig. 1 in context of type-2 fuzzy relations.

with the membership function:

$$\mu_{\tilde{R}}(x_i, y_j) \in [0, 1].$$

For examples for type-1 fuzzy relations, see footnote 3.

Now we can introduce the concept of interval type-2 fuzzy relation. This concept is very important, as it will be employed in establishing uncertain and imprecise relationships between medical entities.

Definition 6 (*Interval type-2 fuzzy relation*) Let $X = \{x_1, x_2, x_3, ..., x_n\}$ and $Y = \{y_1, y_2, y_3, ..., y_m\}$ be referential sets. An interval type-2 fuzzy relation, denoted $\tilde{\tilde{R}}$ on $X \times Y$, is defined:

$$\tilde{\tilde{R}} : X \times Y \to \mathcal{F}([0, 1]),$$

where $\mathcal{F}([0, 1])$ represents the set of all subintervals of the interval [0,1]:

$$\mathcal{F}([0, 1]) = \{[x_L, x_U] : x_L,\ x_U \in [0, 1],\ x_L \leq x_U\},$$

$$\tilde{\tilde{R}} = \left\{(x_i, y_j), \left[\mu_{\tilde{R}}(x_i, y_j)_L, \mu_{\tilde{R}}(x_i, y_j)_U\right]\right\},$$

with the primary membership function:

$$\mu_{\tilde{R}}(x_i, y_j)_L, \mu_{\tilde{R}}(x_i, y_j)_U \in [0, 1]$$

and,

$$\forall (x_i, y_j), \mu_{\tilde{R}}(x_i, y_j)_L \leqslant \mu_{\tilde{R}}(x_i, y_j)_U$$

representing the lower and upper bound of $\tilde{\tilde{R}}$ elements respectively.

Based on Definitions 6 and 4, a type-2 fuzzy relation interval value is characterized by specific lower and upper boundaries instead of a fuzzy set, as is the case in type-2 fuzzy relations (Fig. 1). As all values of the secondary membership function equal 1, the uncertainty is represented by associated intervals.[6]

A type-2 fuzzy inclusion relation is characterized by a fuzzy set (Fig. 2).

[6] An example of an interval type-2 relation: s: "increased serum glucose" and d: "diabetes":

$$(s \xleftarrow[1]{} d, s \xrightarrow[{[0.6,0.7]}]{} d);$$

s always occurs with d but it only confirms d with certain possible values within [0.6,0.7].

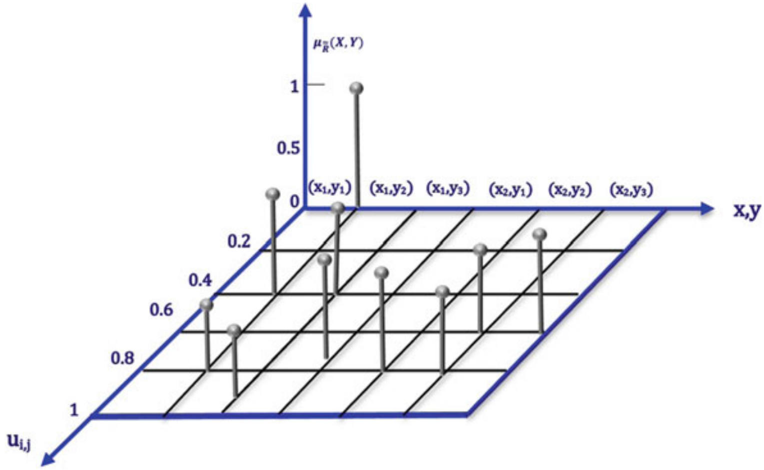

Fig. 1 Interval type-2 fuzzy relation (Definition 6)

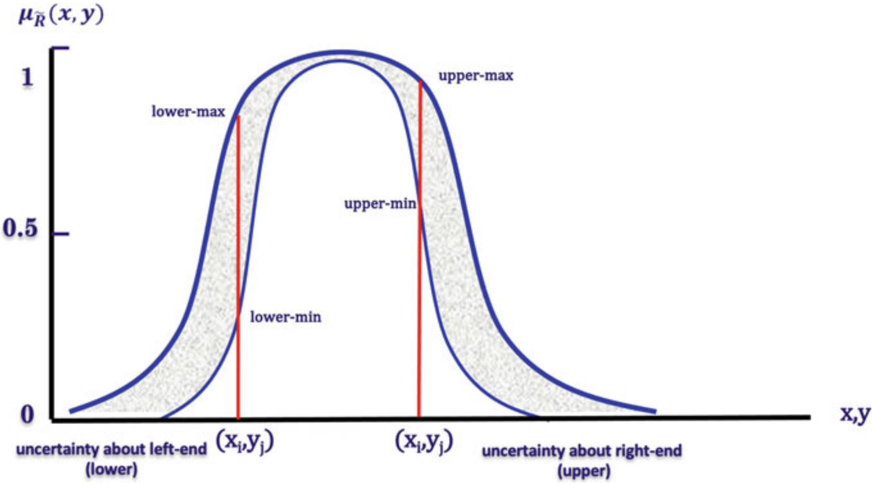

Fig. 2 Type-2 fuzzy relation. The footprint of uncertainty (FOU) is represented by the lower-min and upper-max of possible uncertain degrees of a type-2 fuzzy relation. Each end shows some uncertainties; we use lower-min and lower-max for the left-end, upper-min and upper-max for the right-end uncertainties

2 Associative Medical Relationships

Representing medical entities as fuzzy sets and establishing type-1 or type-2 fuzzy inclusion relationships among them provides us with a *framework* to represent uncertain medical knowledge. Binary fuzzy inclusion or the subsethood measure describes the degree to which a fuzzy set is included in another. In other words, it expresses the degree of subsethood relation between two fuzzy sets [27]. This kind of inclusion is useful to express the degree of association between medical entities represented as fuzzy sets. The necessity of occurrence and the sufficiency of occurrence between fuzzy medical entities covers the most important aspects of establishing an *associative relationship* between different medical entities. These aspects can be interpreted as the degree to which a medical entity is implied in another. Furthermore, considering interval type-2 fuzzy relations relying on an inclusion measure enables us to consider the uncertainty and vagueness between associative medical entities.

In the following we will present interval-valued fuzzy relations relying on the degree of subsethood to model the uncertainty and imprecision:

Definition 7 (*Type-1 fuzzy inclusion relation, $\tilde{R}_{\mathcal{I}}$*) Let $\tilde{A}$ and $\tilde{B}$ be fuzzy subsets of $\mathcal{U} = \{x_1, x_2, ..., x_n\}$. A type-1 fuzzy inclusion relation, denoted $\tilde{R}_{\mathcal{I}}$, is defined as

$$\tilde{R}_{\mathcal{I}} : \mathcal{F}(\mathcal{U}) \times \mathcal{F}(\mathcal{U}) \to [0, 1],$$

where $\mathcal{F}(\mathcal{U})$ represents the set of all fuzzy sets in $\mathcal{U}$,

$$\tilde{R} = \left\{ \left((\tilde{A}, \tilde{B}), \mu_{\tilde{R}_{\mathcal{I}}}(\tilde{A}, \tilde{B}) \right) \right\}$$

with

$$\mu_{\tilde{R}_{\mathcal{I}}}\left(\tilde{A}, \tilde{B}\right) \triangleq \frac{\sum\limits_{x \in \mathcal{U}} \min\left(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\right)}{\sum\limits_{x \in \mathcal{U}} \mu_{\tilde{A}}(x)} \in [0, 1] \; and \tag{3}$$

$$the\; scaler\; cardinality\; of\; A, |A| = \sum_{x \in \mathcal{U}} \mu_{\tilde{A}}(x) \neq 0.$$

Scalar inclusion measure expresses to which degree a fuzzy set is included in another one; i.e.

$$\mu_{\tilde{R}_{\mathcal{I}}}(\tilde{A}, \tilde{B}) \triangleq degree(\tilde{A} \subseteq \tilde{B}) \in [0, 1] \tag{4}$$

and,

$$\mu_{\tilde{R}_{\mathcal{I}}}(\tilde{B}, \tilde{A}) \triangleq degree(\tilde{B} \subseteq \tilde{A}) \in [0, 1] \tag{5}$$

These relations can be interpreted in terms of CADIAG-II as *strength of confirmation* (μ_c):

$$(s \underset{\mu_c}{\longrightarrow} d),\ with\ \mu_c \in [0, 1]$$

and *frequency of occurrence* (μ_o):

$$(s \overset{\mu_o}{\longleftarrow} d),\ with\ \mu_o \in [0, 1]$$

or *sufficiency* and *necessity* respectively.

Definition 8 (*Interval type-2 fuzzy inclusion relation, $\tilde{\tilde{R}}_I$*) The uncertainty of a type-2 fuzzy inclusion relation, denoted $\tilde{\tilde{R}}_{\mathcal{I}}$, is associated with intervals given by type-1 fuzzy relation:

$$\tilde{\tilde{R}}_{\mathcal{I}} : \mathcal{F}(\mathcal{U}) \times \mathcal{F}(\mathcal{U}) \to \mathcal{F}([0, 1]),$$

where $\mathcal{F}([0, 1])$ represents the set of all subintervals of the interval [0,1]:

$$\mathcal{F}([0, 1]) = \{[x_L, x_U] : x_L,\ x_U \in [0, 1],\ x_L \leq x_U\},$$

such that

$$\tilde{\tilde{R}}_I = \left\{\left((\tilde{A}, \tilde{B}), \left[\mu_{\tilde{R}_I}(\tilde{A}, \tilde{B})_L, \mu_{\tilde{R}_{\mathcal{I}}}(\tilde{A}, \tilde{B})_U\right]\right)\right\},$$

$$\mu_{\tilde{R}_I}(\tilde{A}, \tilde{B})_L \leqslant \mu_{\tilde{R}_{\mathcal{I}}}(\tilde{A}, \tilde{B})_U$$

The interval $[\mu_{\tilde{R}_I}(\tilde{A}, \tilde{B})_L, \mu_{\tilde{R}_{\mathcal{I}}}(\tilde{A}, \tilde{B})_U]$ expresses the certain possible consistent degrees of the scalar inclusion relationship between $\tilde{A}$ and $\tilde{B}$, see Fig. 7.

Notably, a type-2 fuzzy inclusion relation interval value is characterized by specific lower and upper boundaries, while a *type-2 fuzzy inclusion relation* is characterized by a fuzzy set (Fig. 7 vs. Fig. 3).

Definition 9 (*Uncertain associative medical relationships*) Let $\mathcal{E} = \{e_1, e_2, e_3, ..., e_n\}$ be a set of medical entities represented as fuzzy sets. Uncertain associative relationship between medical entities can be interpreted as type-2 fuzzy inclusion relation.

The focus of this presentation will be on interval type-2 fuzzy relation, $\tilde{\tilde{R}}_I$:

$$\tilde{\tilde{R}}_{\mathcal{I}} : \mathcal{F}(\mathcal{E}) \times \mathcal{F}(\mathcal{E}) \to \mathcal{F}([0, 1]).$$

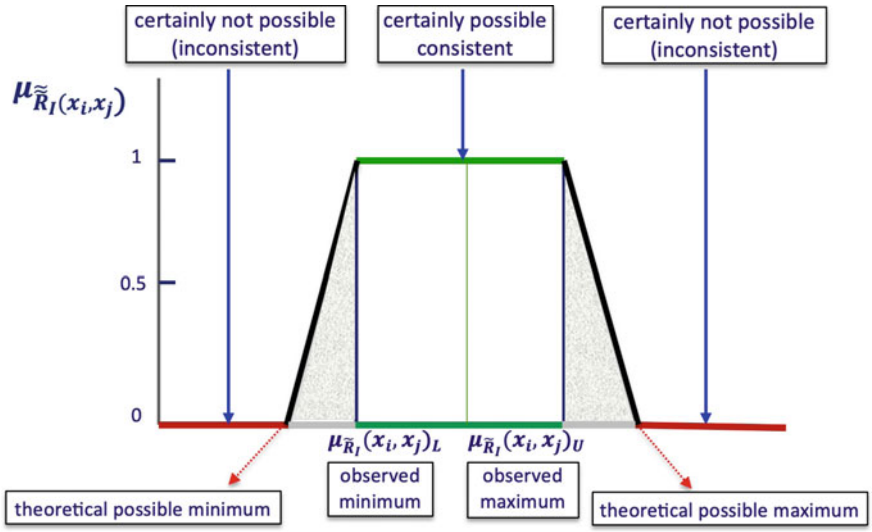

Fig. 3 General type-2 fuzzy inclusion relation. Values of type-1 fuzzy inclusion relation are characterized by type-2 inclusion relation. Values between observed minimum and maximum are certain

2.1 Acquiring Data for Associative Medical Relationships

Two major approaches may be used for dealing with medical knowledge acquisition, namely the classical knowledge-driven approach (symbolic representation in the context of linguistic uncertainty and imprecision), and the data-driven approach. The latter has been given greater importance in recent years, as we are approaching the era of big data and deep learning. Concrete data for instantiating associative medical relationships can be obtained from a variety of sources:

- Evaluating linguistic documentations by medical experts [13].
- Statistical analyses of medical patient databases [12].
- Data discovery in medical databases, i.e. utilizing data science methods on patient databases and documentations, such as predictive classification or descriptive methods (e.g., associative rule analysis), Fig. 4.

Domain experts cannot always deliver precise and consistent values for associative relationships without evaluating a *large quantity of medical data*. For example, a symptom that always occurs in a certain disease might *not be sufficient* to confirm the disease. One example of strong relationships would be *"highly increased amylase levels almost confirm acute pancreatitis"*. This type of associative relationship can be represented by considering a compound an interval type-2 fuzzy relation (see example in footnote 6).

The process of refinement of such intervals should be concluded by *checking* them for *consistency*. It should be noted that global consistency might refine the

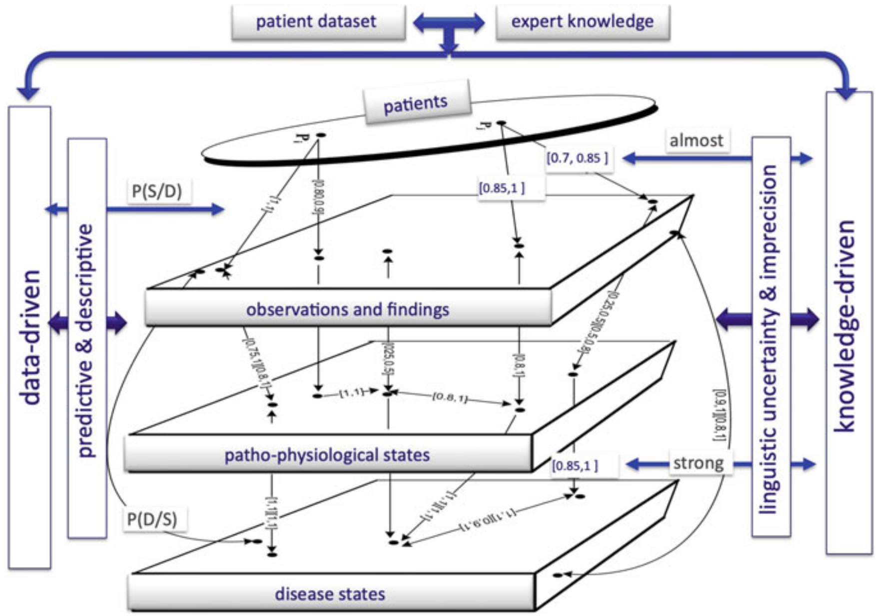

Fig. 4 Medical context of knowledge-driven and data-driven approaches for capturing uncertainty at the medical knowledge acquisition process. Interval-valued uncertainty is present at all levels of knowledge acquisition. In this Figure, the uncertain relationships such as (s $\xrightarrow[strong]{}$ d) can be expressed in terms of interval type-2 fuzzy relation, formalized as (s $\xrightarrow[[0.85,1]]{}$ d)

upper and *lower* values, so that useful global *minima* might be acquired. Establishing such uncertainties requires a *stepwise knowledge acquisition* process and refinement; some cases are provided in Fig. 5. In such processes, an associative relationship might start with no prior knowledge or may be a simple association, such as a *positive or negative correlation*, and might end with a type-1 fuzzy relation, (Definition 1, $\tilde{R}_I$) or a *consistent interval* (Definition 8, $\tilde{\tilde{R}}_I$). In each step or phase, the expert may add knowledge that would refine the degree of imprecision and uncertainty. However, as associative relationships in all phases might be affected by some degree of uncertainty, useful inferential knowledge can be successively added to the acquisition process.

Furthermore, *initial values* can be estimated *statistically* by analyzing a medical database. This approach has been successfully employed as semi-automatic knowledge acquisition within the knowledge-based system CADIAG-II/Rheuma [3, 12]. In this context, necessity (*frequency of occurrence*) may be interpreted as P(S/D), and sufficiency (*strength of confirmation*) as P(D/S), which might be estimated via Bayes' theorem and refined or transformed to fuzzy values.

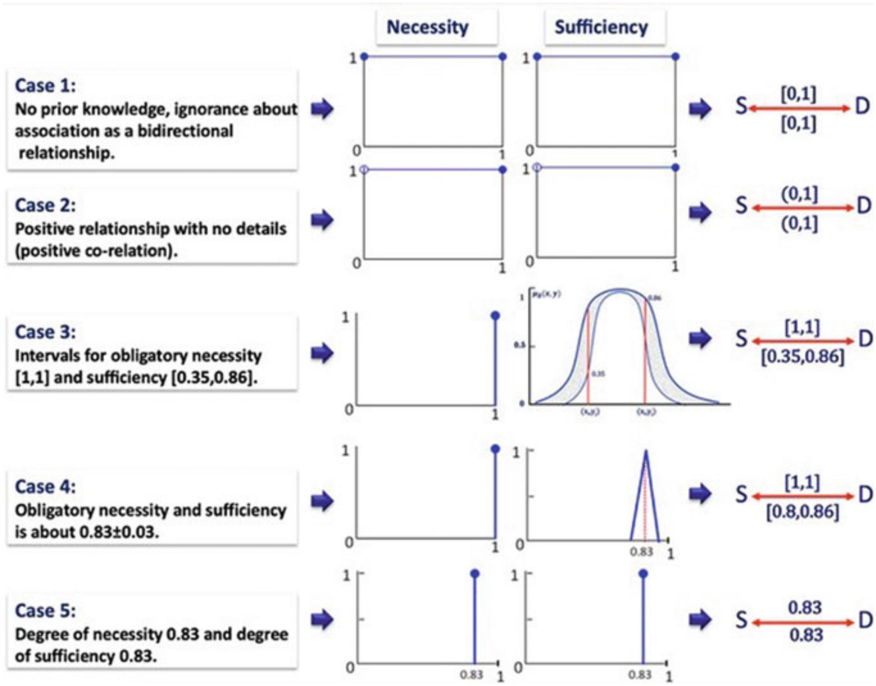

Fig. 5 Example of representing medical uncertainty in the context of necessity and sufficiency of occurrence between medical entities based on the concept of interval type-2 fuzzy relation. The boundaries of FOU, footprint of uncertainty, can be reduced by a refinement process checking the boundaries for local and global consistency

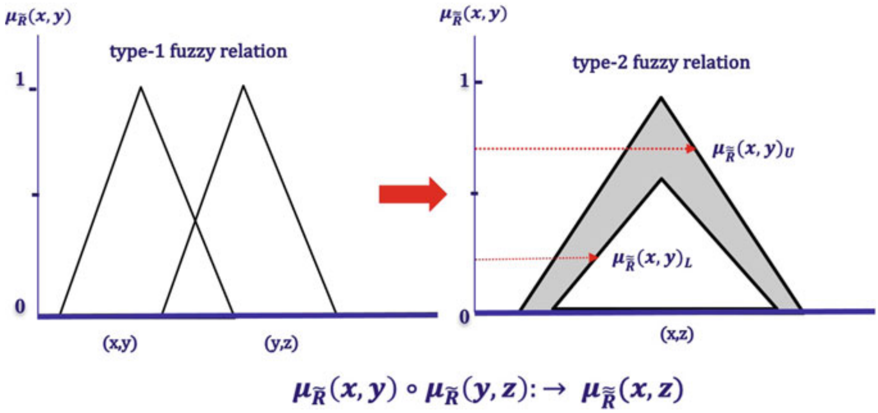

Fig. 6 The composition of type-1 binary fuzzy relations results in a type-2 relation when enhancing it with uncertainty by a local triangular dataset representation

3 Inferencing Type-2 Fuzzy Relation

As mentioned earlier, a variety of sources may be used to acquire concrete data for instantiating associative medical relationships, such as evaluating linguistic documentations, statistical analysis of medical patient databases, or data-driven tasks. However, the creation of knowledge bases with a large number of relationships between medical entities might result in inconsistencies and incompleteness. Furthermore, in many cases, decision-making under imprecision and uncertainty is required.

Several human domain experts might suggest inconsistent estimations of associative relationships in the context of *relevance estimation and assessment*. In some cases, an *agreement* or *disagreement analysis* of the involved experts should be considered. The grade of agreement or disagreement or bias might be used as a reference for considering the degree of uncertainty [14].

To access this important aspect, we need an inferential model that is capable of computing all possible consistent values for a type-1 and even type-2 fuzzy relation. Values lying outside these intervals should be considered as inconsistent values (Fig. 7). Systems affected by inconsistency might reduce the performance of a knowledge-based system.

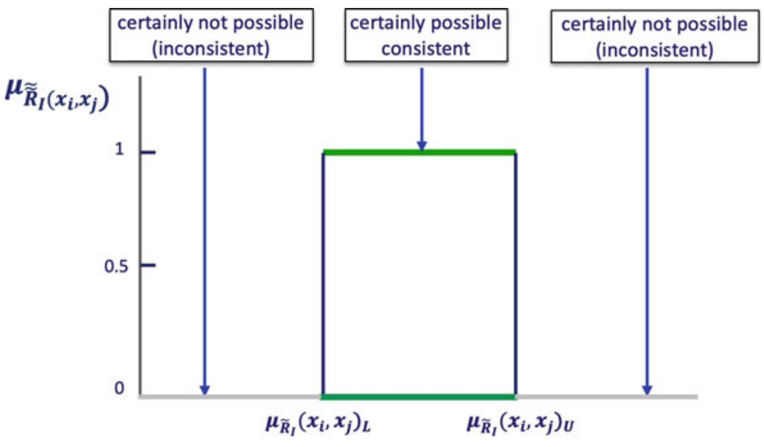

Fig. 7 Interval type-2 fuzzy relation representing an interval-valued binary fuzzy relationship. The certain possible values with $\mu_{\tilde{\tilde{R}}_I}(x_i, y_j) = 1$ are consistent. The final goal is to compute the interval of such type-2 inclusion fuzzy relations

3.1 Inferential Uncertain Relations

This illustration (Fig. 7) extends from the inconsistent interval of uncertainty to certainly not possible values for uncertainties. The present paper will be limited to introducing the *basic concept* of the *inferential model* in the case of type-1 fuzzy relation.[7] This type has been widely used in the different implementations of CADIAG-II-based knowledge-based systems [5–11].

Notably, the computed consistent intervals might be interpreted as boundaries for possible uncertainties arising from assuming the certainty of precise values such as $\mu_{\tilde{A}}(x)$. Figure 6 shows that the composition of two type-1 fuzzy relations, i.e. certain fuzzy relations, would propagate uncertainty in form of consistent intervals.[8]

In the following, we will focus on the basic case of inferring consistent intervals within locally investigated triangular datasets to infer consistent intervals and their minima for the *upper* and *lower* boundaries; i.e. FOU. In this model, we differentiate between local and global uncertainty.

Let $\mathcal{M}$ be a triangular dataset of medical entities consisting of point-valued type-1 relations (Definitions 5, 7), $\tilde{R}_{\mathcal{I}}$:

$$\begin{aligned}\mathcal{M} = \{e_1 \xrightarrow[a_2]{} e_2, e_1 \xleftarrow{a_1} e_2,\\ e_2 \xrightarrow[b_2]{} e_3,\\ e_2 \xleftarrow{b_1} e_3\}\end{aligned}$$

The possible consistent type-1 fuzzy relationships between e_1 and e_3 can be computed as *interval type-2 relation*, i.e.:

$$\mathcal{M} \Vdash \{e_1 \xrightarrow[[\underline{x}_1, \bar{x}_1]]{} e_3, e_3 \xleftarrow[[\underline{x}_2, \bar{x}_2]]{} e_1\}$$

The interval $[\underline{x}_i, \bar{x}_i]$ represents the lower and upper boundaries for uncertainty; i.e.:

$$\begin{aligned}\underline{x}_1 &= \mu_{\tilde{R}_I}(e_1, e_3)_L\\ \bar{x}_1 &= \mu_{\tilde{R}_I}(e_1, e_3)_U\\ \underline{x}_2 &= \mu_{\tilde{R}_I}(e_3, e_1)_L\\ \bar{x}_2 &= \mu_{\tilde{R}_I}(e_3, e_1)_U,\end{aligned}$$

[7] Considering all other aspects, such type-2 inference exceeds the scope of the current presentation.

[8] Obligatory sufficiency and necessity of yield certainty, refer to Fig. 6, i.e.:

$$\mu_{\tilde{R}_{\mathcal{I}}}(x, y) = 1, \mu_{\tilde{R}_{\mathcal{I}}}(y, x) = 1, \mu_{\tilde{R}_{\mathcal{I}}}(y, z) = 1, \; and \; \mu_{\tilde{R}_{\mathcal{I}}}(z, y) = 1$$

and they are computed as follows:

$$\underline{x}_1 = \begin{cases} a_2 - \dfrac{a_2}{a_1} \cdot (1 - b_2) & \text{if } b_2 > 1 - a_1, a_1 \neq 0 \\ 0 & otherwise \end{cases} \tag{6}$$

$$\bar{x}_1 = \begin{cases} \min\left(a_2, \left(\dfrac{b_2 \cdot a_2}{a_1}\right)\right) + \min\left(1 - a_2, \left(\left(\dfrac{b_2 \cdot a_2}{b_1 \cdot a_1}\right) \cdot\ (1 - b_1)\right)\right) \\ \qquad\qquad\qquad\qquad \text{if } b_1 \neq 0, a_1 \neq 0 \\ 0 & \text{if } b_1 = 1, a_1 = a_2 = 0 \\ 1 - a_2 & otherwise \end{cases} \tag{7}$$

$$\underline{x}_2 = \begin{cases} \underline{x}_1 \cdot \left(\dfrac{b_1 \cdot a_1}{b_2 \cdot a_2}\right) & \text{if } b_2 \neq 0, a_2 \neq 0 \\ 0 & otherwise \end{cases} \tag{8}$$

$$\bar{x}_2 = \begin{cases} \bar{x}_1 \cdot \left(\dfrac{b_1 \cdot a_1}{b_2 \cdot a_2}\right) & \text{if } b_2 \neq 0, a_2 \neq 0 \\ 0 & \text{if } b_1 = 0, b_2 = 0, a_1 = 1 \\ 1 - b_1 & otherwise \end{cases} \tag{9}$$

The derivation of these formulae can be achieved by considering all possible inclusion degrees within the triangular dataset $\mathcal{M}$; such as $degree(e_1 \subset e_2)$, $degree(e_2 \subset e_1)$, $degree(e_2 \subset e_3)$, and $degree(e_3 \subset e_2)$ in context of computing the minimal and maximal $degree(e_1 \subset e_3)$, and $degree(e_3 \subset e_1)$. All these relationships can be expressed in terms of constraints represented as computable linear equalities and/or inequalities. Solving all these constraints in reference to $|e_1| = \sum\limits_{x \in \mathcal{U}} \mu_{e_1}(x)$ yields an interval of possible degrees for $(e_1 \supset e_3)$ and $(e_3 \supset e_1)$. The basic idea of the derivation can also be found in [27].

Example 1 Let $\mathcal{M}$ be a triangular set of relations of a type-1 fuzzy relation; Definition 5:

$$\begin{aligned} \mathcal{M} = \{e_1 \underset{0.5}{\longrightarrow} e_2, e_1 \overset{0.75}{\longleftarrow} e_2, \\ e_2 \underset{1}{\longrightarrow} e_3, \\ e_2 \overset{0.25}{\longleftarrow} e_3\} \end{aligned}$$

Based on the Eqs. 6 and 7, all instances of possible relationships are:

$$(e_1 \xrightarrow[{[\underline{x}_1, \bar{x}_1]}]{} e_3) \in \mathcal{S},$$

where $\mathcal{S}$:

$$\mathcal{S} = \{(e_1 \xrightarrow[{[\underline{x}_1, \bar{x}_1]}]{} e_3) | [\underline{x}, \bar{x}] \subseteq [0.5, 1]\} \tag{10}$$

are consistent instances with $\mathcal{M}$ in terms of uncertainty.

The computation of globally consistent intervals requires inference of minimal intervals globally, which can be implemented *incrementally* and *recursively*.

3.2 Application Potential

As mentioned earlier, this approach is founded on the following aspects:

- Employing interval type-1 and type-2 fuzzy relations expressing the necessity and sufficiency of occurrence in the context of establishing associative relationships between medical entities; e.g., point-valued, linguistic and interval-valued:

  $$(s \xrightarrow[\mu]{} d, s \xrightarrow[strong]{} d, s \xrightarrow[{[a,b]}]{} d)$$

  The basic concepts were *partly* employed in designing a CADIAG-II-based system such as MedFrame/CADIAG-IV, and implemented as a stepwise incremental refinement acquisition system [13].
- Inferencing useful consistent intervals to refine and possibly derive new associative relationships, and to check the knowledge base for logical inconsistencies.[9]
- Finally, integrating the compositional rule of inference within this model in connection with an inference engine within a decision support system.

The following example illustrates the application potential of integrating the inference model into refinement and data quality assurance:

Example 2 Let $\mathcal{M}$ be a triangular dataset of compound relationships as defined in Example 1. Based on 3.1, the inferred relationships:

$$(e_1 \xrightarrow[{[0.5,1]}]{} e_3)$$

and

[9] The authors are working on integrating the introduced inference model within the stepwise refinement process. However, further research will be needed to consider complex relationships expressing logical combinations of medical entities on the left side of a rule in the context of the global and local consistency.

$$(e_1 \xleftarrow[{[0.187,0.375]}]{} e_3)$$

can be represented as a consistent bi-directional (compound) interval-valued type-2 relationship, (Definition 8)

$$e_1 \xleftrightarrow[{[0.50,1]}]{[0.187,0.375]} e_3. \tag{11}$$

Such relationships are very useful for the following tasks:

- Checking an relationship for logical consistency (10). For instance, the rule in (12) describes a possible relationship expressing the degree of sufficiency and it is consistent with $\mathcal{M}$ with some certainly possible values:

$$e_1 \xrightarrow[{[0.75,0.95]}]{} e_3, \tag{12}$$

  while the relationship in (13):

$$e_1 \xrightarrow[{[0.25,0.345]}]{} e_3 \tag{13}$$

  represents an inconsistent relationship, as the values exceed the scope of the certainly possible values, see Fig. 7.
- The relationship between e_1 and e_3 in (11); e.g., $e_1 \xrightarrow[{[0.50,1]}]{} e_3$, and $(e_1 \xleftarrow[{[0.187,0.375]}]{} e_3)$ can be added to the knowledge base to increase some issues related to performance and completeness.
- Under the assumption that previous knowledge has already been validated on consistency, this approach relies on consistent interval propagation. In case an expert would propose new values for $(e_i \rightarrow e_j)$, the new values are expected to lie within the certainly possible values of the computed formula. However, further interval refinement is possible by considering new knowledge. The refinement process (i.e. narrowing the fingerprint of certainty) can be achieved by computing the global consistency of the model under the newly added values.
- Finally, integrating the compositional rule of inference within this inference, we can follow an inference engine within a decision support system.

4 Conclusion and Future Perspectives

This paper describes the handling of some crucial aspects of knowledge representation, relying on the acquisition of consistent inferential associative relationships. The adopted approach emphasizes the importance of considering fuzzy sets and type-2 relations with a view to the establishment of associative medical relationships within an inferential model, considering uncertainty, and checking for logical consistency. Many aspects of this model have been successfully employed by different implementations of CADIAG-II-based systems. For future work, the integration of this approach within a stepwise refinement of the knowledge acquisition process will be significant for ensuring data quality and enhancing performance. Furthermore, an interval-based compositional rule of inference might lead to a form of reasoning that relies on inferencing consistent uncertain intervals. Finally, it would be desirable to fine-tune the established intervals by integrating data-driven approaches. Clustering, relationships, and interval-valued-based deep learning might be useful approaches.

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